**Theorem** (Fermat)**.** *Let* $p \in \mathbb{Z}^+$ *with* $p \equiv 1 \pmod 4$ *be a prime. Then: there are* $x, y \in \mathbb{Z}$ *with* $x^2 + y^2 = p$*.*

*Proof.* Let $\chi_4 : \mathbb{F}_p^\times \twoheadrightarrow \mu_4 \subseteq \mathbb{C}^\times$, and note that $J := \sum_{x \in \mathbb{F}_p} \chi_4(x)\chi_4(1-x) \in \mathbb{Z}[i]$ has $|J|^2 = p$.