Theorem. *Let f be a bounded entire function. Then: f is constant.*

Proof. Write $g(z) := f(z) - f(0) =: \sum_{n\in\mathbb{Z}^+} a_n \cdot z^n$. But $||g||^2_{L^2(\{|z|=R\})} = \sum_{n\in\mathbb{Z}^+} |a_n|^2 \cdot R^{2n}$. Thus all $a_n = 0$.